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ANOTHER PROOF OF A THEOREM IN MULTIPLY PERFECT NUMBERS

BY R. D. CARMICHAEL

J. WESTLUND has shown[*] that the only multiply perfect numbers of multiplicity 3 and of the form $p_1^{a_1} p_2^{a_2} p_3$, where p_1, p_2, p_3 are different primes and $p_1 < p_2 < p_3$, are $2^3 \cdot 3 \cdot 5$ and $2^5 \cdot 5 \cdot 7$. I have shown[†] that there is none of multiplicity 3 and of the form $p_1^{a_1} p_2^{a_2} p_3^{a_3}$, where $a_3 > 1$ and p_1, p_2, p_3 are as before. It is proposed now to give a very simple demonstration of the latter theorem. For this purpose we require to use a theorem employed in my paper on "Multiply perfect numbers of four different primes."[‡] This proposition is as follows:

If x *is a positive integer greater than* 1, $x^t - 1$ *has a prime factor not dividing* $x^u - 1 (u < t)$, *except in the two cases* $t = 2$, $x = 2^v - 1$, $v \geqq 2$; *and* $t = 6$, $x = 2$. *Such prime factors of* $x^t - 1$ *are of the form* $st + 1$, *and evidently if* t *is odd and greater than* 1 *they are of the form* $2st + 1$.

For the problem in consideration we must have[§]

$$3 = \frac{2^{a_1+1} - 1}{2^{a_1}} \cdot \frac{3^{a_2+1} - 1}{3^{a_2}.2} \cdot \frac{p_3^{a_3+1} - 1}{p_3^{a_3}(p_3 - 1)},$$

if such multiply perfect numbers exist. But, by the theorem above quoted, if $a_3 > 1$, $p_3^{a_3+1} - 1$ must have a prime factor greater than 3, since the exceptional cases are excluded by the conditions imposed upon p_3 and a_3. Therefore the equation cannot be satisfied. Hence,

There are no multiply perfect numbers of multiplicity 3 *and of the form* $p_1^{a_1} p_2^{a_2} p_3^{a_3}$, *where* $p_1 < p_2 < p_3$ *are different primes and* $a_3 > 1$.

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[*] ANNALS OF MATHEMATICS, ser. 2, vol. 2, p. 172 (1901).

[†] Ibid., vol. 7, p. 153 (1906), and vol. 8, p. 49 (1906).

[‡] Ibid., vol. 8, p. 151 (1907).

[§] Ibid., vol. 7, p. 153 (1906).